

RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)
B.A./B.Sc. THIRD SEMESTER EXAMINATION, MARCH 2022
SECOND YEAR [BATCH 2020-23]

Date : 09/03/2022

MATHEMATICS (General)

Time : 11am-1pm

Paper : MAGT 1

Full Marks : 50

Group A

Answer any three questions of the following:

[3 × 6=18]

1. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel straight lines, then show that the distance between them is $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$. [6]
2. Find the points of contact and equations of tangents to the conic $x^2 + y^2 + 6x - 10y - 15 = 0$ which are parallel to x - axis. [6]
3. Find the equation of $y^2 = 8x$ in the polar form with the vertex as the pole and the positive direction of the x - axis as the positive direction of the initial line. [6]
4. Show that the length of the focal chord of the conic $\frac{l}{r} = 1 - e \cos \theta$ which is inclined to the initial line at an angle α is $\frac{2l}{1 - e^2 \cos^2 \alpha}$. [6]
5. Find the equation of a right circular cone which passes through the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and whose axis is $\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$. [6]

Group B

Answer any four questions of the following:

[4 × 8=32]

6. (a) Prove that if n is an integer and θ is a real number,
 $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$. [5]
(b) Find the cube roots of $-1 - i$. [3]
7. (a) Find the principal value of $(1 - i)^i$. [3]
(b) Prove that if $f(x)$ is a polynomial with real coefficients and $\alpha + i\beta$, ($\beta \neq 0$) be a root of $f(x) = 0$, then $\alpha - i\beta$ will be another root of $f(x) = 0$. [5]
8. (a) Solve the equation $x^4 + 6x^3 + 13x^2 + 12x - 5 = 0$. Given that the sum of the two roots is equal to the sum of the other two. [4]
(b) If α, β, γ be the roots of the equation $2x^3 - 9x^2 + 7x + 6 = 0$, find the equation whose roots are $\alpha^2 + \beta^2 - \gamma^2, \beta^2 + \gamma^2 - \alpha^2, \gamma^2 + \alpha^2 - \beta^2$. [4]
9. (a) Check whether the following set of vectors is linearly dependent or not. If yes, then express one of the vectors as linear combination of others.
 $(1, 0, 0, 1), (1, 1, 1, 0), (0, 1, 1, 1), (1, 1, 1, 1)$. [5]
(b) State Cayley-Hamilton theorem. Use it find A^{-1} , where $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. [3]
10. (a) Find the eigen values and corresponding eigen vectors of the matrix $B = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$. [5]
(b) Let (G, \circ) be a cyclic group, generated by a . Prove that a^{-1} is also a generator. [3]

11. (a) Let (G, \circ) be a group and H be a non empty subset of G . Prove that (H, \circ) forms a subgroup of (G, \circ) if and only if $a \in H, b \in H \implies a \circ b^{-1} \in H$. [5]
- (b) Define divisors of zero in a ring. Give an example of a finite ring with divisors of zero. [2+1]

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